liquid and considering the latter to be Newtonian, we use overstated theoretical values of its flow velocity in microcapillaries. In actuality, the "equivalent" viscosity of such a liquid in microcapillaries is higher than handbook values of its shear viscosity [2]. As a result, despite the fact that Φ^{mp} is determined not only by the linear velocity but also by microrotation, their total contribution to Φ^{mp} is less than Φ^{n}, which is determined only by the linear-velocity gradient — since the theoretical values of the latter are higher than the actual values.

Consequently, allowing for the natural rotations of particles of a micropolar liquid leads to a substantial (with corresponding values of the microstructural parameters k and δ_0, as well as ξ) reduction in the theoretical values of its dissipative heating in the region of stabilized heat exchange.

NOTATION

2h, distance between plates; dp/dx, pressure gradient; v_x and ν_z, components of the velocity and microrotation vectors; μ, κ, β, and γ, material constants of the micropolar liquid; α, boundary-condition parameter; Φ, dissipative function; t_{kl}, stress tensor; m_{kl}, micromoments tensor; λ_t and λ_q, thermal conductivities of the materials of the channel and liquid; T, temperature; H_1 and H_2, thicknesses of channel walls; T_c, temperature of outside surfaces of channel; e_{rkl}, antisymmetric tensor; $\vec{w} = (1/2)\,\mathrm{rot}\,\vec{v}$, vorticity vector.

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THERMAL INTERACTION BETWEEN A PIPELINE AND THE SURROUNDING FROZEN GROUND

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A method is proposed for computing heat-transfer processes of pipelines and other engineering structures with finely dispersed frozen ground.

The exploitation of pipelines under low-temperature conditions of the surrounding ground is fraught with numerous complications. Reduction of the temperature of the product being transported can result in elevation of its viscosity (for oil), formation of ice (for water), and hydrated locks (for gases). Warming up the surrounding ground results in disturbance of its stability and, as a result, in pipeline buckling and undesirable ecological consequences. The most unfavorable are the pipeline exploitation conditions during its startup, when thermal losses are especially large. This same period is most complex from the viewpoint of the methodology of thermal design since nonstationary effects must be taken into account. These complexities grow significantly when the pipeline is in finely dispersed soils in which the phase transitions extend into the temperature spectrum.

Features of the thermal interaction between a pipeline and finely dispersed frozen soil are analyzed in this paper.

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The process of starting to exploit a pipeline is examined. The ground in which the pipeline is laid is in the frozen state up to the starting time. We neglect the presence of snow cover on the ground surface. If necessary, its influence can be taken into account by using the method of the "additional layer." The temperature of the liquid in the pipeline is assumed constant and positive. We write the heat-conduction problem for the surrounding finely dispersed ground by assuming that all the interstitial liquid is connected, undergoing phase transitions according to a known law in the negative temperature spectrum [1, 2]:

$$\rho c_{ef}\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial\bar{x}}\left(\lambda\frac{\partial\theta}{\partial\bar{x}}\right)+\frac{\partial}{\partial\bar{y}}\left(\lambda\frac{\partial\theta}{\partial\bar{y}}\right); \tag{1}$$

$$\bar{y}\geqslant 0;\ \bar{x}^2+(\bar{y}-H)^2>R_0^2;\ c_{ef}=c-l\omega\frac{dW}{d\theta};$$

$$\theta|_{\tau=\tau_0}=T_n(\bar{y},\ \tau_0); \tag{2}$$

$$\lambda\frac{\partial\theta}{\partial\bar{y}}\bigg|_{\bar{y}=0}=\alpha_a(\theta-T_a)_{\bar{y}=0}; \tag{3}$$

$$\lambda\frac{\partial\theta}{\partial\bar{r}}\bigg|_{\bar{r}=R_0}=\alpha(\theta-T_0)_{\bar{r}=R_0}; \tag{4}$$

$$\theta|_{\bar{x}\to\infty}=T_n(\bar{y},\ \tau); \tag{5}$$

$$\theta|_{\bar{y}\to\infty}=T_n(\infty,\ \tau). \tag{6}$$

Here W is the relative weighted iciness of the ground [3], which can be described by the following dependence [4]:

$$W=\begin{cases}0, & \theta>0,\\ 1-\dfrac{1}{1-A\theta}; & \theta\leqslant 0;\end{cases} \tag{7}$$

ω is the total moisture [3]; c_{ef} is the effective specific heat of the ground with the effect of molecular heating (cooling) and liberation (absorption) of the latent heat of the phase transitions. The magnitude of the parameter A characterizes the shape of the iciness curve and is determined by the type of ground and the moisture. The thermophysical characteristics of the soil vary with temperature mainly because of the change in iciness:

$$\lambda=\lambda_t+(\lambda_f-\lambda_t)W;\ c=c_t+(c_f-c_t)W. \tag{8}$$

For simplicity, the linear dependence of λ and c on the iciness is taken here for boundary values corresponding to the frozen and thawed states of the ground. Only conductive heat transport is taken into account in the system (1)-(8). We neglect the effects of migration of the interstitial liquid and vapor.

As an analysis of the solution of the corresponding problem without phase transitions shows [5, 6], during the considerable time interval from the time of pipeline startup, the solution of this problem differs slightly from the axisymmetric case. This is explained by the fact that the influence of the daytime surface on the heat-transfer process starts to be felt substantially after a certain time associated with the time of thermal perturbation passage from the pipeline to the daytime surface. This circumstance permits considering the solution of the axisymmetric problem as the standard, close to the solution of the two-dimensional problem in the initial time interval.

Meanwhile, an algorithm of the numerical solution has been worked up for one-dimensional axisymmetric and plane-parallel problems, which was later applied to the solution of the two-dimensional problem. This is caused, firstly, by the great simplicity of the one-dimensional problems and, secondly, by the existence of exact and approximate solutions in a number of cases, with which the obtained results are compared.

The heat-conduction problem in soil has the following form in an axisymmetric formulation:

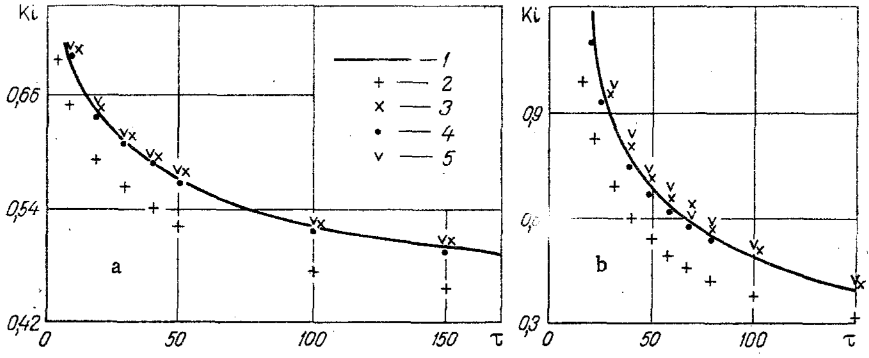

Fig. 1. Dependence of the Kirpichev criterion Ki on time for axisymmetric (a) and plane-parallel (b) problems. Stefan problem (1); A = 0.5, ω = 0.208 (2); A = 4.5, ω = 0.208 (4); A = 0.5, ω_{re} = 0.208 (3); A = 4.5, ω_{re} = 0.208 (5); τ, h.

$$\rho c_{ef}\frac{\partial\theta}{\partial\tau}=\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\lambda\frac{\partial\theta}{\partial\bar{r}}\right);\ R_0<\bar{r}<\infty; \tag{9}$$

$$\lambda\frac{\partial\theta}{\partial\bar{r}}\bigg|_{\bar{r}=R_0}=\alpha(\theta-T_0)_{\bar{r}=R_0}; \tag{10}$$

$$\theta|_{\tau=\tau_0}=T_n(H,\ \tau_0)=T_f\ . \tag{11}$$

To obtain the numerical solution of this problem, we go from the semiinfinite domain of definition [R_0, ∞) to the finite domain [1, 0] by using a transformation of the independent variable:

$$\xi=R_0/\bar{r}. \tag{12}$$

This substitution reduces the problem (9)-(11) to the following:

$$\rho c_{ef}\frac{\partial\theta}{\partial\tau}=\frac{\xi^3}{R_0^2}\frac{\partial}{\partial\xi}\left(\lambda\xi\frac{\partial\theta}{\partial\xi}\right); \tag{13}$$

$$\frac{\lambda}{R_0}\frac{\partial\theta}{\partial\xi}\bigg|_{\xi=1}=\alpha(T_0-\theta)_{\xi=1}; \tag{14}$$

$$\theta\Big|_{\substack{\xi=0\\ \tau=\tau_0}}=T_f\ . \tag{15}$$

which is replaced by a system of finite-difference equations whose solution is sought by the method of factorization.

The algorithm obtained as the result was realized on an EC LIPSE C/300 computer for which a program was compiled in the algorithmic language FORTRAN V. Computations were performed for the following initial data: T_f = −1°C, T_0 = 25°C, λ_t = 1.54 kcal/(m•h•deg), λ_f = 1.8 kcal/(m•h•deg), A = 0.5 deg^{-1} and 4.5 deg^{-1}, l = 80 kcal/kg, c_t = 0.442 kcal/(kg•deg), c_f = 0.44 kcal/(kg•deg), R_0 = 0.14 m, α = 6.05 kcal/(m^2•h•deg), and ω = 0.208. The value A = 0.5 corresponds to clayey soil, and A = 4.5 to sand.

We analyze the results of the computations by the magnitude of the Kirpichev criterion

$$\mathrm{Ki}=-\frac{1}{2\pi(T_0-T_f)}\int_0^{2\pi}R_0\frac{\partial\theta}{\partial\bar{r}}\bigg|_{\bar{r}=R_0}d\varphi, \tag{16}$$

which is governing in the computation of the pipeline temperature regime since it characterizes the heat losses in the soil.

Graphs of the dependence Ki(τ) are presented in Fig. 1a for two considered values of A corresponding to soils of different disperseness. The dependence Ki(τ) is represented in this same figure for the case A = ∞ (Stefan problem) obtained by reducing the initial problem to a system of integral equations [7, 8].

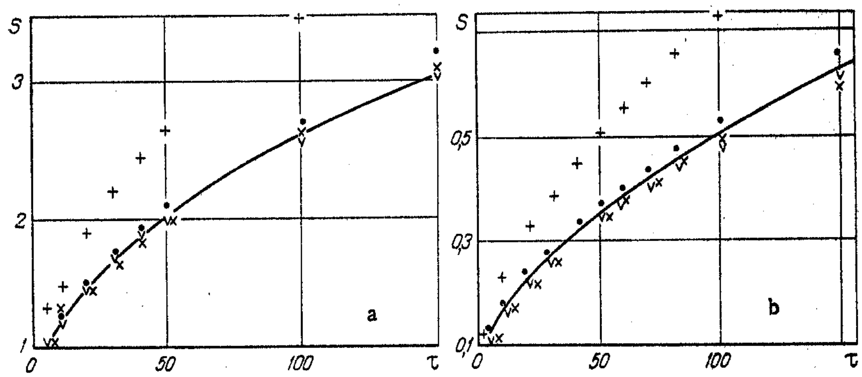

Fig. 2. Change in the zeroth isotherm coordinate with time for the axisymmetric (a) and plane-parallel (b) problems. Notation the same as in Fig. 1.

Curves of $S(\tau)$, the coordinates of the zeroth isotherm, computed for the cases considered, are represented in Fig. 2a.

As is seen from Figs. 1a and 2a, for an identical total moisture the function $Ki(\tau)$ diminishes while the function $S(\tau)$ grows more rapidly for soils with a smaller value of the parameter A (i.e., with a more diffuse iciness spectrum). Curves corresponding to the Stefan problem ($A = \infty$) correspond to the same dimensionality. Let us note the following circumstance. Soils of different disperseness have a different initial relative iciness for an identical initial temperature T_f:

$$W_f = 1 - \frac{1}{1 - AT_f}. \tag{17}$$

Consequently, for an identical total moisture ω their ice content (i.e., the ice mass per unit volume of soil) is different at the initial instant. Moreover, the change in the ice content for an identical change in temperature is also different.

Taking into account the fact that the main contribution to the effective specific heat of the soil is the heat of the ice–water phase transition, the difference between the curves $Ki(\tau)$ and $S(\tau)$ described above can be explained by the difference in the effective specific heats of the soils of different disperseness for an identical total moisture.

Let us introduce the concept of reduced moisture ω_{re}, the moisture under Stefan problem conditions that will assure the same heat transfer characteristics as in the problem with a spectrum of a phase transition with the moisture ω. The relationship between these quantities under axisymmetric problem conditions will be sought from the following integral equality:

$$\int_0^{T_0} \omega_{re} (T_0 - T)\, dT = \int_0^{T_0} \omega W_f (T_0 - T)\, dT + \int_{T_f}^{0} \omega (W_f - W)(T_0 - T)\, dT.$$

It is an approximate expression of the condition of an identical change in the ice content for the Stefan problem and the problem with a phase-transition spectrum. The factors $(T_0 - T)$ are added to the integrand to take account of the axisymmetry of the temperature field.

We hence obtain

$$\omega_{re} = \omega \left\{ 1 - \frac{1}{1 - AT_f} + \frac{2}{AT_0^2} \left[\left(T_0 - \frac{1}{A} \right) \ln (1 - AT_f) - T_f \right] \right\}. \tag{18}$$

In particular, $\omega_{re} = \omega$ for the Stefan problem.

Curves of $Ki(\tau)$ and $S(\tau)$ are presented in Figs. 1a and 2a for soils with parameters A = 0.5 and 4.5 of the iciness spectrum and moistures assuring a value $\omega_{re} = 0.208$ for the reduced moistures. As is seen from these figures, the identical values of the reduced moistures assure the practical merger of these curves with each other and with the curves for the Stefan problem. Analogous features are also traced in the solution of the one-dimensional plane-parallel problem. Let us examine it in the following formulation:

$$\rho c_{ef}\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial\bar{x}}\left(\lambda\frac{\partial\theta}{\partial\bar{x}}\right),\quad 0\leqslant\bar{x}<\infty; \tag{19}$$

$$\theta|_{\bar{x}=0}=T_0; \tag{20}$$

$$\theta|_{\tau=0}=T_f. \tag{21}$$

Results of a numerical solution of this problem are represented in Figs. 1b and 2b for values of the parameter A = 0.5 and 4.5 and the total moisture $\omega = 0.208$. Also presented here are data for the exact solution of this problem for $A = \infty$ (Stefan problem) [9]. As is seen from the graphs, the thermal fluxes and coordinates of the zeroth isotherm are related for different A exactly as in the axisymmetric problem.

We seek the magnitude of the reduced moisture from the relation

$$\int_0^{T_0}\omega_{re}\,dT=\int_0^{T_0}\omega W_f\,dT+\int_{T_f}^{0}\omega(W_f-W)\,dT.$$

We hence obtain for the plane-parallel case

$$\omega_{re}=\omega\left\{1-\left(1-\frac{T_f}{T_0}\right)\frac{1}{(1-AT_f)}+\frac{1}{AT_0}\ln(1-AT_f)\right\}. \tag{22}$$

The results of computing the heat-transfer parameters of soils with different disperseness and identical reduced moisture ($\omega_{re} = 0.208$), represented in Figs. 1b and 2b, are practically coincident.

Therefore, solutions of the Stefan problem can be used to compute heat-transfer processes for pipelines and other engineering structures with frozen finely dispersed soils by replacing the true by the reduced moisture in the initial data. This circumstance considerably facilitates the computation since a significant number of approximate analytic methods exist for solving the Stefan problem, which are used successfully in design and research practice.

On the other hand, realization of the numerical solution of the problem with a phase-transition spectrum is simpler than the Stefan problem. This is associated with the fact that the effective specific heat becomes infinite in the latter at the phase transition temperature, while the thermophysical characteristics undergo a discontinuity.

Let us turn to the two-dimensional problem (1)-(6). Following [6, 10], we apply the following conformal mapping for its numerical solution:

$$\bar{x}+i\bar{y}=R_0c_0 i\,\mathrm{cth}\left(\frac{\alpha_0x_1+\pi x_2 i}{2}\right). \tag{23}$$

Here $c_0 = \sqrt{h^2-1}$, $\alpha_0 = \ln(h + c_0)$, $h = H/R_0$. This transformation reduces problem (1)-(6) to the following form:

$$\rho c_{ef}\frac{\partial\theta}{\partial\tau}=\frac{1}{R_0^2a}\left[\frac{1}{\alpha_0^2}\frac{\partial}{\partial x_1}\left(\lambda\frac{\partial\theta}{\partial x_1}\right)+\frac{1}{\pi^2}\frac{\partial}{\partial x_2}\left(\lambda\frac{\partial\theta}{\partial x_2}\right)\right], \tag{24}$$

$$0\leqslant x_1\leqslant 1;\ 0\leqslant x_2\leqslant 1;$$

$$\lambda\frac{\partial\theta}{\partial x_1}=\frac{\alpha\alpha_0R_0c_0(T_0-\theta)}{h-\cos(\pi x_2)}\quad\text{for}\quad x_1=1; \tag{25}$$

$$\lambda\frac{\partial\theta}{\partial x_1}=\frac{\alpha_a\alpha_0R_0c_0(\theta-T_a)}{1-\cos(\pi x_2)}\quad\text{for}\quad x_1=0; \tag{26}$$

$$\frac{\partial\theta}{\partial x_2}=0\quad\text{for}\quad\begin{cases}x_2=0;\\x_2=1;\end{cases} \tag{27}$$

$$\theta = T_e \quad \text{for} \quad \tau = \tau_0. \tag{28}$$

Here

$$a = \frac{c_0^2}{[\mathrm{ch}\,(\alpha_0 x_1) - \cos(\pi x_2)]^2}.$$

Problem (24)-(28) was solved by using a locally one-dimensional difference scheme [11, 12]. In conformity with this method, the two-dimensional equation (24) is separated into two one-dimensional equations

$$\frac{\rho c_{ef}}{2}\frac{\partial v_1}{\partial \tau} = \frac{1}{R_0^2 a \alpha_0^2}\frac{\partial}{\partial x_1}\left(\lambda \frac{\partial v_1}{\partial x_1}\right); \tag{29}$$

$$\frac{\rho c_{ef}}{2}\frac{\partial v_2}{\partial \tau} = \frac{1}{R_0^2 a \pi^2}\frac{\partial}{\partial x_2}\left(\lambda \frac{\partial v_2}{\partial x_2}\right). \tag{30}$$

The boundary conditions on the functions v_1 and v_2 are:

$$\lambda \frac{\partial v_1}{\partial x_1} = \frac{\alpha \alpha_0 R_0 c_0 (T_0 - v_1)}{h - \cos(\pi x_2)} \quad \text{for} \quad x_1 = 1; \tag{31}$$

$$\lambda \frac{\partial v_1}{\partial x_1} = \frac{\alpha_a \alpha_0 R_0 c_0 (v_1 - T_a)}{1 - \cos(\pi x_2)} \quad \text{for} \quad x_1 = 0; \tag{32}$$

$$\frac{\partial v_2}{\partial x_2} = 0 \quad \text{for} \quad \begin{array}{l} x_2 = 0; \\ x_2 = 1. \end{array} \tag{33}$$

The system is integrated in each spacing in time (τ^j, τ^{j+1}) under the following initial conditions:

$$v_1(x_1, x_2, \tau^j) = \theta(x_1, x_2, \tau^j); \tag{34}$$

$$v_2(x_1, x_2, \tau^j) = v_1(x_1, x_2, \tau^{j+1}); \tag{35}$$

$$\theta(x_1, x_2, \tau^{j+1}) = v_2(x_1, x_2, \tau^{j+1}). \tag{36}$$

The thermal interaction of a pipeline with the surrounding frozen soil was computed from the algorithm elucidated for $H = 1.9$ m and $R_0 = 0.216$ m. The thermophysical contents were assumed the same as for the axisymmetric problem.

As follows from the results of the computations, during a long period after startup, the magnitudes of the thermal fluxes on the tube wall $\mathrm{Ki}(\tau)$ are practically coincident for the two-dimensional and the axisymmetric problems. Thus, in the problem considered, a substantial difference from the axisymmetric solution starts only for $\tau = 1200$ h, i.e., after 50 days. Such a nature of the function $\mathrm{Ki}(\tau)$ permits application of a method analogous to one proposed earlier for a single-phase problem [5]:

$$\mathrm{Ki} = \max\left\{\frac{1}{\alpha_0};\ \frac{T_0}{(T_0 - T_f)(\ln S + 1/\alpha_t)}\right\}$$

for its solution. Here the quantity S is determined in conformity with [13] for the total moisture value $\omega = \omega_{re}$, $\alpha_t = R_0\alpha/\lambda_t$, i.e., in the initial period the quantity Ki is found by one of the approximate methods for the axisymmetric two-phase problem (in this case, in conformity with [13]), and later, after the solution emerges at a value determined for the stationary regime by the Forschheimer formula, is taken equal to the corresponding expression.

In conclusion, we examine the method of selecting the spacing of the spatial partition for the numerical integration of (13). In order to select the optimal partition spacing of the domain of definition of the independent variable ξ [0, 1], a numerical experiment was performed. The problem was solved here for a different quantity of partition points, from 50 to 1000. It was finally obtained that the results are stabilized for N = 500. At the same time, a number of partitions N = 50 assures sufficient accuracy for an analogous single-phase problem. So abrupt an increase in the necessary number of partitions in the case of the two-phase problem is explained by the fact that the effective specific heat grows sharply in the domain of liberation (absorption) of the heat of phase transition.

Taking into account the fact that an abrupt growth in the effective specific heat is localized in a narrow zone of negative temperatures near the zeroth isotherm, it is expedient to use a shallow step in space only within the limits of this zone. This permits a significant diminution in the dimensionality of the arrays utilized and an increase in the computation rate, which is especially valuable under multidimensional problem conditions. It should be noted that, however, computational difficulties associated with the appearance of a moving zone of condensation of the spatial partition points grow here.

The spatial partition in the square $[0 \leq x_1 \leq 1;\ 0 \leq x_2 \leq 1]$ was selected in the program described above for the solution of the two-dimensional problem, from the following considerations. The nature of the temperature fields v_1 and v_2 into which the initial temperature field is divided under the conditions of a locally one-dimensional scheme is substantially distinct. The function v_1 is characterized by significant gradients and is qualitatively similar to the temperature distribution for the axisymmetric problem. The function v_2 is the correction for nonaxial symmetry of the temperature field and is characterized by small gradients.

Taking this into account, the partitioning along the x_1 axis was executed analogously to the axisymmetric problem with condensation in the zone of intensive phase transitions, and with a uniform coarse spacing along the x_2 axis. Such a method permits the reduction of the dimensionality of two-dimensional arrays with 500×500 to 70×50 (if a uniform partition were executed with a spacing to assure the necessary accuracy).

NOTATION

ρ, soil density; c_{ef}, effective heat conduction; c_t and c_f, specific heats of the thawed and frozen soil; τ, time coordinate; $\bar{x}$ and $\bar{y}$, space coordinates; H, depth of pipeline axis location; l, heat of the phase transition; λ_t and λ_f, heat-conduction coefficients of the thawed and frozen soil; α_a, heat-transfer coefficient from the soil surface in air; α, heat-transfer coefficient from the liquid to the surrounding soil; T_a, air temperature; T_0, product temperature; T_n, natural soil temperature; T_f, initial (frozen) soil temperature; A, parameter of the iciness spectrum; R_0, radius of the outer pipeline surface; Ki, Kirpichev criterion; S, dimensionless radius of the phase transition front.

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